Effective Quarks and Their Interactions[1]

Emili Bagan[2]

Physics Department
Brookhaven National Laboratory
Upton NY 11973
USA

Robin Horan[3], Martin Lavelle[3] and David McMullan[3]

School of Mathematics and Statistics
The University of Plymouth
Plymouth, PL4 8AA
UK

Abstract: This talk will summarise the progress we have made in our programme to both characterise and construct charges in gauge theories. As an application of these ideas we will see how the dominant glue surrounding quarks, which is responsible for asymptotic freedom, emerges from a constituent description of the interquark potential.

Introduction

The phenomenological confidence in the existence of coloured hadronic constituents is in marked contrast to the theoretical uncertainties associated with attempts to describe such charges. Although there are many models of partonic and effective degrees of freedom in the literature, none have yet emerged directly from the fundamental gauge theoretic description of the strong interactions — QCD.

The source of the difficulty in directly extracting these effective degrees of freedom from the underlying gauge theory is a particular example of the basic dichotomy we all face in QCD: the degrees of freedom that make up the QCD Lagrangian and successfully probe the ultra-violet regime are not related in any obvious way to the large scale, infrared degrees of freedom that describe the observed hadrons or their constituents.

The picture that has emerged[4] from studies of deep inelastic scattering, and more recently from diffractive processes, is that, as we probe smaller and smaller sub-hadronic

[1]Talk presented by D. McMullan

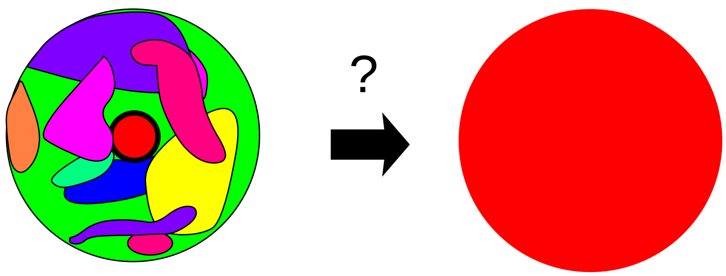
[2]email: bagan@ifae.es

[3]emails: rhoran@plymouth.ac.uk, mlavelle@plymouth.ac.uk and dmcmullan@plymouth.ac.uk

[4]For a nice discussion of the stunning experimental results achieved over the past decade see [1].

scales, we go from wholly hadronic degrees of freedom to constituent structures (quarks). In their turn, these constituents have properties (for example, their mass) that run as shorter distance scales are probed. As such they are not viewed as fundamental fields but as composites made up in some way from the matter and gluonic degrees of freedom that enter the QCD Lagrangian.

There is an immediate theoretical problem for any coloured constituent: how does such a quark or gluon have a well defined colour given that, as is shown in the picture below, it is made up of some mixture of apparently coloured partonic degrees of freedom?



In order to answer this fundamental question and hence introduce our approach [2] to the construction of such (colour) charged degrees of freedom, we need to recall *how* and *when* colour can be defined in QCD.

Colour and Gauge Invariance

The very structure of QCD as a gauge theory tells us that physical fields *must* be invariant under local[5] gauge transformations. This apparent truism generates though an immediate problem when we look at the colour charge itself:

$$Q^a = \int d^3x \, (J_0^a(x) - f^a_{bc} E_i^b(x) A_i^c(x)) \, . \tag{1}$$

This is clearly not gauge invariant! So can we talk about colour in any meaningful way? The answer [3] can be seen to be yes, when we recognise that the question we should be asking is whether the colour charge is gauge invariant when restricted to physical (i.e., gauge invariant) states. On these states, the non-abelian version of Gauss' law implies that

$$Q^a = \frac{1}{g} \int d^3x \, \partial_i E_i^a(x) \, . \tag{2}$$

[5]In QCD we actually expect the stronger statement to emerge that physical fields are invariant under *all* gauge transformations: both local and global, in other words colour charges are supposed to be confined in (hadronic) colour singlets. We will see at the end of this article how this stronger result actually emerges directly from our approach to the constructions of colour charges.

Under a gauge transformation $E_i^a T^a \to U^{-1} E_i^a T^a U$ so that

$$Q^a T^a \to \frac{1}{g} \int d^3x \, \partial_i (U^{-1} E_i^a T^a U) \, . \tag{3}$$

We can now write this as the surface integral of the chromo-electric flux in any given direction and hence we see that, on gauge invariant states, the colour charge transforms as

$$Q^a T^a \to \frac{1}{g} \lim_{R \to \infty} \int_{S^2} d\underline{s} \cdot U^{-1} \underline{\mathcal{E}} U \, . \tag{4}$$

Hence the colour charge will be gauge invariant under local gauge transformations if, at spatial infinity, we have $U \to U_\infty$ where U_∞ lies in the centre of SU(3). Continuity then tells us that this group element will be a constant independent of the direction taken to spatial infinity. This imposes a $\mathbb{Z}_3$ (triality) structure on possible charged states. However, we will only concern ourselves with the zero triality sector in this talk where U_∞ is the identity. To summarise the above discussion, we have seen that in order to be able to define a coloured object, such as a quark, we require that it must be gauge invariant and also that allowed gauge transformations must be restricted as above. We now want to study the construction of charged fields.

Construction of Charges

In QCD we have the gauge transformations

$$\begin{aligned} A_\mu(x) &\to U^{-1}(x) A_\mu(x) U(x) + \frac{1}{g} U^{-1}(x) \partial_\mu U(x) \\ \psi(x) &\to U^{-1}(x) \psi(x) \end{aligned}$$

From the non-triviality of these transformations we see that neither of these fields have a well defined colour and hence they *cannot* be identified with observed gluonic or quark degrees of freedom.

The generic form for a charged (matter) field is given by a process we call *dressing* the matter to give the product

$$h^{-1}(x) \psi(x) \, , \tag{5}$$

where, under a gauge transformation, the dressing transforms as

$$h^{-1}(x) \to h^{-1}(x) U(x) \, . \tag{6}$$

This is the minimal condition we must impose on the dressing in order for the corresponding charged matter field to be gauge invariant and hence having a well defined colour [2]. But does this really means we have a physical field? Is gauge invariance alone enough? Consider the stringy gauge invariant e^+e^- state

$$|\bar{\psi}(x) \exp(ie \int_x^y dw A(w)) \psi(y)\rangle \tag{7}$$

This is gauge invariant, but is it physical? For static matter, the energy is the expectation value of the Hamiltonian $\frac{1}{2}\int d^3z(E^2(z)+B^2(z))$. This yields for the potential energy of this state the confining potential

$$V(x-y)\sim e^2|x-y|\delta^2(0)\,. \tag{8}$$

Given that we are here dealing with four dimensional QED, it is clear that the original stringy state cannot be accepted as a physical configuration! Indeed it is an infinitely excited state and what we would want to construct is the ground state for the system. To do this in a systematic fashion, we need a further condition apart from gauge invariance. The question which we now address is: what condition on the dressing gives a stable charged particle?

In order to motivate this extra condition, we first consider φ to be a heavy field which creates a particle at the point x with a given 4-velocity u. The field must be constant along the trajectory of a particle moving with this velocity which leads to the equations of motion:

$$u^\mu\partial_\mu\varphi(x)=0\,. \tag{9}$$

If φ is now a heavy gauged field then its equation of motion becomes

$$u^\mu D_\mu\varphi(x)=0\,. \tag{10}$$

A physical heavy coloured field can only emerge from dressing this field, $\phi=h^{-1}\varphi$. If this is truly a heavy field, then it should furthermore satisfy the equation

$$u^\mu\partial_\mu\phi(x)=0\,. \tag{11}$$

This means that the dressing must satisfy the *dressing equation*

$$u^\mu\partial_\mu(h^{-1})=gh^{-1}u^\mu A_\mu\,. \tag{12}$$

One can in fact show that this equation applies to any theory with massive charges [4].

Electric Charges

It is instructive to first study the dressing process in the simpler case of QED. Our two inputs into the construction are then, as we have just seen, gauge invariance $h^{-1}\to h^{-1}\mathrm{e}^{-ie\theta}$ and our kinematical requirement (the dressing equation)

$$u^\mu\partial_\mu(h^{-1})=-ieh^{-1}u^\mu A_\mu \tag{13}$$

The great advantage of QED is that we can explicitly solve [5] these equations to obtain

$$h^{-1}=\mathrm{e}^{-ieK}\mathrm{e}^{-ie\chi}\,, \tag{14}$$

where

$$K(x)=-\int_\Gamma d\Gamma(\eta+v)^\mu\frac{\partial^\nu F_{\nu\mu}}{\mathcal{G}\cdot\partial}\,, \tag{15}$$

$$\chi(x)=\frac{\mathcal{G}\cdot A}{\mathcal{G}\cdot\partial}\,, \tag{16}$$

with $\eta=(1,\underline{0})$, $v=(0,\underline{v})$, $\mathcal{G}^\mu=(\eta+v)^\mu(\eta-v)\cdot\partial-\partial^\mu$ and where Γ is the trajectory of the particle.

We thus see that the dressing has two structures: a gauge dependent part, χ, which makes the whole charge gauge invariant, and is thus in some sense minimal, and a further gauge invariant part, K which is needed (together with the precise form of χ) to satisfy the dressing equation. These structures are reflected in physical calculations: χ removes soft divergences in QED calculations and, as we will see, in QCD generates the anti-screening interaction responsible for asymptotic freedom. K removes the phase divergences in the on-shell Green's functions of QED[6].

For greater insight into these structures, let us consider the specific case of a static charge, $v=0$. This is:

$$h^{-1}\psi(x)=\mathrm{e}^{-ieK}\mathrm{e}^{-ie\chi}\psi(x) \tag{17}$$

with now

$$K(x)=-\int_{-\infty}^{x^0}dt\frac{\partial^\nu F_{\nu 0}}{\nabla^2} \tag{18}$$

$$\chi(x)=\frac{\partial_i A_i}{\nabla^2} \tag{19}$$

and where

$$\frac{1}{\nabla^2}f(t,\underline{x}):=-\frac{1}{4\pi}\int d^3y\frac{f(t,\underline{y})}{|\underline{x}-\underline{y}|} \tag{20}$$

The non-locality of any description of a charge is manifest here. The minimal part part of the electromagnetic cloud around a static charge was first found by Dirac [7]. The additional structure does not affect the electric field of the charge and was therefore not picked up by Dirac's original argument. We will now show that such charged (dressed) matter is free at large times and that we can so recover a particle description [8].

To see why this is important, we now recall that Kulish and Faddeev [9] showed that, at large times the matter field is not free, but rather becomes[7]

$$\psi(x)\to\int\frac{d^3p}{(2\pi)^3}\frac{D(p,t)}{\sqrt{2E_p}}\left\{b(\boldsymbol{p},s)u^s(\boldsymbol{p})e^{-ip\cdot x}+\cdots\right\}, \tag{21}$$

where D is a *distortion factor*. This implies that there is no particle picture. Of course since the coupling does not asymptotically vanish ψ is not gauge invariant even at large times and so we should not expect to relate it to a physical particle! However, when we extract the annihilation operator for our *dressed* field we obtain

$$b(q)\left\{1+e\int_{\text{soft}}\frac{d^3k}{(2\pi)^3}\left(\frac{V\cdot a}{V\cdot k}-\frac{q\cdot a}{q\cdot k}\right)\mathrm{e}^{-itk\cdot q/E_q}-\text{c.c.}\right\}+O(e^2), \tag{22}$$

with $V^\mu=(\eta+v)^\mu(\eta-v)\cdot k-k^\mu$.

There are two corrections now: the usual one [9] from the interactions of the matter field and another one from the dressing. It is easy to show [8] that at the right point on

[6]The cancellation of the various IR divergences [6] will be presented in the talk by Martin Lavelle.

[7]For more details of this method and a refinement of their work, see the talk by Robin Horan

the mass shell, $q = m\gamma(\eta + v)$, these distortions cancel! We thus see that our dressed matter asymptotically corresponds to free fields and we regain a particle picture.

Colour Charges

After this construction of abelian charges, we now want to proceed to the non-abelian theory. We recall that the *minimal* static dressing in QED was: $\exp(-ie\chi)$, with $\chi = \partial_i A_i/\nabla^2$. This vanishes in Coulomb gauge and this observation lets us generalise this dressing to QCD where it may be extended to an arbitrary order in g (see the Appendix of [2]. Indeed we can also extend to non-static charges, but in the application that follows we require static quarks.

In QCD we write the dressing as a perturbative expansion

$$\exp(-ie\chi) \Rightarrow \exp(g\chi^a T^a) \equiv h^{-1} \tag{23}$$

with $g\chi^a T^a = (g\chi_1^a + g^2\chi_2^a + g^3\chi_3^a + \cdots)T^a$

The dressing gauge argument mentioned above implies

$$\chi_1^a = \frac{\partial_j A_j^a}{\nabla^2}\,; \quad \chi_2^a = f^{abc}\frac{\partial_j}{\nabla^2}\left(\chi_1^b A_j^c + \frac{1}{2}(\partial_j \chi_1^b)\chi_1^c\right) \tag{24}$$

etc. This can be extended to all orders in the coupling. However, we will return below to the question of non-perturbative solutions.

The Interquark Potential

We now want to study the interaction energy of the ground state in the presence of a matter field and its antimatter equivalent [10]. To construct this we can either dress a quark and an antiquark separately or dress a single meson in which there are no gauge invariant constituents. We will now dress the quark fields and study the potential between them: whether or not this gives the correct interaction energy, i.e., the potential, is a test of the validity of a constituent picture. We recall from our discussion of the 'stringy' state that this is a sensitive test.

Our procedure is as follows: as sketched above we first extend our expression for the minimally dressed quark to higher orders in perturbation theory. We then take such minimally dressed quark/antiquark states, $\bar{\psi}(y)h(y)h^{-1}(y')\psi(y')|0\rangle$, and sandwich the Hamiltonian,

$$H = \frac{1}{2}\int (E_i^a E_i^a + B_i^a B_i^a)d^3x \tag{25}$$

between them. Using the standard equal-time commutators

$$[E_i^a(x), A_j^b(y)]_{\text{et}} = i\delta^{ab}\delta(\boldsymbol{x} - \boldsymbol{y})\,, \tag{26}$$

we can then calculate the potential.

The lowest order result, i.e., at order g^2, is just the Coulomb potential:

$$V^{g^2}(r) = -\frac{g^2 N C_F}{4\pi r}\,, \tag{27}$$

where r is the separation of the matter fields. This is of course just QED with coloured icing.

What about QCD with non-abelian ingredients? Well at order g^4 we need to calculate the minimal static dressing to order g^3. This can be done with the above mentioned efficient algorithm. A relatively simple calculation then yields for the potential at g^4

$$V^{g^4}(r) = -\frac{g^4}{(4\pi)^2}\frac{N C_F C_A}{2\pi r} 4\log(\mu r)\,. \tag{28}$$

What does this tell us about our dressed state?

We recall that the QCD potential [11] may be extracted from a Wilson loop as follows:

$$V(r) = -\lim_{t\to\infty}\frac{1}{it}\log\langle 0|\mathrm{Tr}\,\mathsf{P}\exp\left(g\oint dx_\mu A^\mu_a T^a\right)|0\rangle \tag{29}$$

At order g^4 this yields

$$V(r) = -\frac{g^2 C_F}{4\pi r}\left[1+\frac{g^2}{4\pi}\frac{C_A}{2\pi}\left(4-\frac{1}{3}\right)\log(\mu r)\right]. \tag{30}$$

From this we may read off the universal one-loop beta function

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[4-\frac{1}{3}\right]. \tag{31}$$

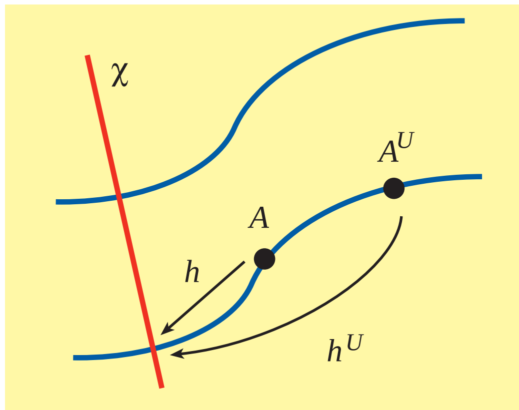
We have decomposed it in this way because it has been shown by a number of authors [12, 13, 14, 15, 16] that the dominant anti-screening contribution (the 4) which is responsible for asymptotic freedom comes from longitudinal glue and the screening part (the $\frac{1}{3}$) from gauge invariant glue

We now recognise that the result (28) for the interaction energy of the minimally dressed quark/antiquark system is just the anti-screening contribution to the interquark potential. We thus make the important identification [10] that the *dominant part of the glue in a $Q\bar{Q}$ system which is responsible for anti-screening actually factorises into two individually gauge invariant constituents*. The success of constituent models can, to the extent that these low order calculations have been carried though, be explained by our work. We postulate that the introduction of gauge invariant glue, via the incorporation of the phase dressings, will produce the screening effect.

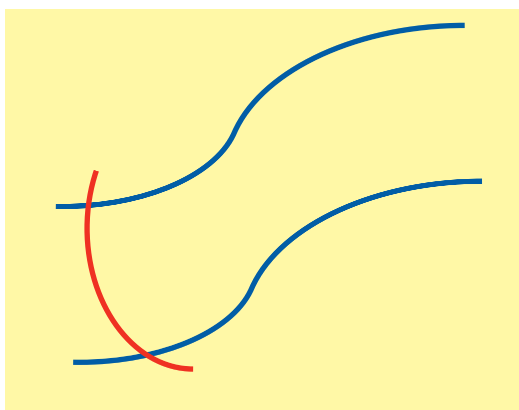
Topology and Confinement

Having seen the perturbative efficiency and relevance of our variables, we now want to study the non-perturbative sector. Experimentally of course we do not see free quarks, and it can be easily shown that this is in fact predicted by our method.

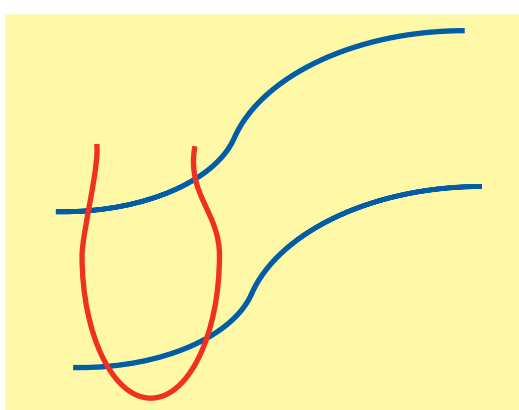
This follows from an intimate link between dressings and gauge-fixing. In the limited space available this can be best explained pictorially. Essentially the existence of a gauge fixing, χ, can be shown [2] to imply the existence of a function h which transforms as a minimal dressing must to make a quark gauge invariant



Similarly we can show [2] that having a dressing implies that we can construct a gauge fixing which slices every gauge orbit once:



But Gribov and Singer [17, 18] have showed that, with certain boundary conditions which we have seen above are needed if colour is to be a good quantum number, there is no such good gauge fixing in QCD!



Hence any constituent picture must break down! We conclude that *there is no non-perturbative, gauge invariant description of a single quark or gluon.* Therefore outside of certain dynamical domains we cannot expect to see individual quarks or gluons.

Conclusions

There are strong phenomenological reasons in both QED and QCD for wanting to be able to describe charged particles. We saw [2] that if we want to describe coloured sub-structure then this requires both gauge invariance and some restrictions on the allowed gauge transformations.

We have seen a method of constructing dressed charges which was designed to describe physical charges with a well defined velocity. It had two inputs: gauge invariance and a kinematical requirement to single out which of the potentially many gauge invariant constructions involving a single matter field corresponds to the ground state. This approach, we have shown, generates structured dressings around charged particles.

It has long been argued that it is impossible to describe charged particles in gauge theories [9]. Essentially this is because asymptotic matter fields are not free fields due to the long range nature of the interactions transmitted by massless gauge bosons. However, we have demonstrated [8] that asymptotic dressed fields are indeed free fields (the dressing takes such effects into account). We thus have obtained for the very first time a particle description of charges. It will be shown elsewhere in this meeting that this removes [6] the IR divergences in QED at the level of on-shell Green's functions.

We have calculated the potential between two minimally dressed quark fields. The anti-screening contribution to the interquark potential was shown to be generated by the minimal dressing. We have thus determined the dominant glue configuration around quarks. We have seen that in a 'meson' – at least to the level at which we calculate – two separate, gauge invariant, coloured objects are visible.

Furthermore there is a topological obstruction to the construction of coloured charges. This means that we can directly demonstrate the non-observability of individual quarks or gluons: outside the domain of perturbation theory and some non-perturbative effects, the Gribov ambiguity will show itself and there will be no locally gauge invariant description of such objects. This shows a new way to calculate the scale of confinement: we need to find out at which stage it becomes impossible to factorise the dressing of, say, a $Q\bar{Q}$ state into two individual charges. Beyond this breakdown of the factorisation quarks do not exist in QCD.

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